OCR
RECOGNISING ACHIEVEMENT

Friday 13 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4756 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions in Section A and **one** question from Section B.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = 1 + \cos\theta$ for $0 \leqslant \theta < 2\pi$.

(i) Sketch the curve. **[2]**

(ii) Find the area of the region enclosed by the curve, giving your answer in exact form. **[6]**

(b) Assuming that x^4 and higher powers may be neglected, write down the Maclaurin series approximations for $\sin x$ and $\cos x$ (where x is in radians).

Hence or otherwise obtain an approximation for $\tan x$ in the form $ax + bx^3$. **[6]**

(c) Find $\displaystyle\int_0^1 \frac{1}{\sqrt{1-\frac{1}{4}x^2}}\,\mathrm{d}x$, giving your answer in exact form. **[4]**

2 **(a)** The infinite series C and S are defined as follows.

$$C = 1 + a\cos\theta + a^2\cos 2\theta + \dots,$$

$$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \dots,$$

where a is a real number and $|a| < 1$.

By considering $C + \mathrm{j}S$, show that $C = \dfrac{1 - a\cos\theta}{1 + a^2 - 2a\cos\theta}$ and find a corresponding expression for S. **[8]**

(b) Express the complex number $z = -1 + \mathrm{j}\sqrt{3}$ in the form $r\,\mathrm{e}^{\mathrm{j}\theta}$.

Find the 4th roots of z in the form $r\,\mathrm{e}^{\mathrm{j}\theta}$.

Show z and its 4th roots in an Argand diagram.

Find the product of the 4th roots and mark this as a point on your Argand diagram. **[10]**

3 **(i)** Show that the characteristic equation of the matrix

$$\mathbf{M} = \begin{pmatrix} 3 & -1 & 2 \\ -4 & 3 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

is $\lambda^3 - 5\lambda^2 - 7\lambda + 35 = 0$. **[4]**

(ii) Show that $\lambda = 5$ is an eigenvalue of $\mathbf{M}$, and find its other eigenvalues. **[4]**

(iii) Find an eigenvector, $\mathbf{v}$, of unit length corresponding to $\lambda = 5$.

State the magnitudes and directions of the vectors $\mathbf{M}^2\mathbf{v}$ and $\mathbf{M}^{-1}\mathbf{v}$. **[6]**

(iv) Use the Cayley-Hamilton theorem to find the constants a, b, c such that

$$\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}.$$ **[4]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Define $\tanh t$ in terms of exponential functions. Sketch the graph of $\tanh t$. **[3]**

(ii) Show that $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. State the set of values of x for which this equation is valid. **[5]**

(iii) Differentiate the equation $\tanh y = x$ with respect to x and hence show that the derivative of $\operatorname{artanh} x$ is $\frac{1}{1-x^2}$.

Show that this result may also be obtained by differentiating the equation in part **(ii)**. **[5]**

(iv) By considering $\operatorname{artanh} x$ as $1 \times \operatorname{artanh} x$ and using integration by parts, show that

$$\int_0^{\frac{1}{2}} \operatorname{artanh} x \, dx = \frac{1}{4}\ln\frac{27}{16}.$$ **[5]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The points A(–1, 0), B(1, 0) and P(x, y) are such that the product of the distances PA and PB is 1. You are given that the cartesian equation of the locus of P is

$$\left((x+1)^2 + y^2\right)\left((x-1)^2 + y^2\right) = 1.$$

(i) Show that this equation may be written in polar form as

$$r^4 + 2r^2 = 4r^2\cos^2\theta.$$

Show that the polar equation simplifies to

$$r^2 = 2\cos 2\theta.$$ **[4]**

(ii) Give a sketch of the curve, stating the values of θ for which the curve is defined. **[4]**

(iii) The equation in part **(i)** is now to be generalised to

$$r^2 = 2\cos 2\theta + k,$$

where k is a constant.

(A) Give sketches of the curve in the cases $k = 1$, $k = 2$. Describe how these two curves differ at the pole.

(B) Give a sketch of the curve in the case $k = 4$. What happens to the shape of the curve as k tends to infinity? **[7]**

(iv) Sketch the curve for the case $k = -1$.

What happens to the curve as $k \to -2$? **[3]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.